



Munich Personal RePEc Archive

How accounting accuracy affects DSGE models

Minseong Kim

29 March 2016

Online at <https://mpa.ub.uni-muenchen.de/70356/>

MPRA Paper No. 70356, posted 29 March 2016 15:42 UTC

How accounting accuracy affects DSGE models

Minseong Kim

2016/03/29

Abstract

This paper explores how accounting consistency affects DSGE models. As many DSGE models descended from real business cycle models, I explore a simple labor-only RBC model with an exogenous external sector introduced. The conclusion reached in this paper is that once an external sector is introduced, DSGE models may suffer from accounting inconsistency, unless disequilibrium or some non-orthodox theory of price level, real monetary supply or bonds is accepted.

1 Accounting consistency of a simple labor-only RBC model with exogenous government and without money

The model is the infinite-life representative agent framework. The household obtains utility $u(C_t, N_t)$ at time t , where C_t is consumption and N_t is labor. Total utility of the household is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \quad (1)$$

where β is time preference. In this economy, nominal factor can be ignored, and thus every variable will be a real variable.

$$u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (2)$$

The household has budget constraint as follows:

$$C_t + R_t^{-1} B_t \leq B_{t-1} + W_t N_t + \Pi_t \quad (3)$$

where B_t is bond, R_t is real interest rate, Π_t is dividend received from the firm. One can immediately stop here and notice that for the fixed income in the right-hand side, there is no reason why the household would buy B_t , unless it affects future consumptions. The rest of this section is developed to demonstrate in

the economy specified that buying more B_t does not increase or decrease future consumption and does not increase or decrease future labor quantity. Future consumption and labor quantity are affected only by expected technology A_{t+k} and expected government deficit spending G_{t+k} that are assumed to be money-financed solely (in other words, finance deficit by printing money), instead of being debt-financed. I will assume that g_t is exogenous, but that the government announced the full path of g_t from present to the infinite future. Let the lower-case z of upper-case variables Z represent $z = \log(z)$. The optimality conditions in the log form are:

$$w_t = \sigma c_t + \varphi n_t \quad (4)$$

$$E_t [c_{t+1}] = c_t + \frac{1}{\sigma}(r_t - \rho) \quad (5)$$

where $\rho = -\log \beta$.

Let the firm maximize profit:

$$\Pi_t = Y_t - W_t N_t \quad (6)$$

with

$$Y_t = C_t + G_t = A_t N_t^{1-\alpha} \quad (7)$$

where G_t is government deficit spending, financed through money. I will not consider inflation as price level P_t is assumed to be uniform across sectors. The optimality condition is

$$w_t = a_t - \alpha n_t + \log(1 - \alpha) \quad (8)$$

By log-linearization assumption, assume:

$$y_t = cc_t + gg_t = a_t + (1 - \alpha)n_t \quad (9)$$

$$c_t = \frac{a_t + (1 - \alpha)n_t - gg_t}{c} \quad (10)$$

where c and g are defined around steady-state values.

Labor-market clearing requires:

$$\sigma \left[\frac{a_t + (1 - \alpha)n_t - gg_t}{c} \right] + \varphi n_t = a_t - \alpha n_t + \log(1 - \alpha) \quad (11)$$

$$\left[\frac{\sigma}{c}(1 - \alpha) + \varphi + \alpha \right] n_t = \left[1 - \frac{\sigma}{c} \right] a_t + \frac{\sigma g}{c} g_t + \log(1 - \alpha) \quad (12)$$

$$n_t = \frac{\left[1 - \frac{\sigma}{c} \right] a_t + \frac{\sigma g}{c} g_t + \log(1 - \alpha)}{\frac{\sigma}{c}(1 - \alpha) + \varphi + \alpha} \quad (13)$$

If technology a_t is assumed to be exogenous, but with known future expected values, then y_t is uniquely specified. Since g_t is already known, c_t is already known. Thus, just from knowledge of g_t , present and expected future c_t can

be calculated. Thus it is now established that B_t does not affect real economy. This result is not affected by whether one takes linearization approximation or not.

The problem, then is the following. For accounting consistency,

$$Y_t = C_t + S_t = C_t + G_t \quad (14)$$

is required (which means $Y_t > C_t$ whenever $G_t > 0$), assuming there is no foreign sector and there is no investment (because this economy is labor-only economy). S_t refers to savings in national accounting. Thus $G_t = S_t$. But notice Equation 3, replicated below:

$$C_t + R_t^{-1}B_t \leq B_{t-1} + W_tN_t + \Pi_t$$

We know that

$$Y_t = W_tN_t + \Pi_t \quad (15)$$

This is true by definition. Thus the budget constraint can be re-written as

$$C_t + R_t^{-1}B_t \leq Y_t + B_{t-1} \quad (16)$$

$Y_t + B_{t-1}$ can be considered as available budget. For the fixed budget $Y_t + B_{t-1}$, there is simply no reason why the household would buy B_t , as this would decrease the household's utility. Furthermore, according to the calculation above, C_{t+1} is unaffected by the quantity of B_t . Thus, $B_t = 0$ in equilibrium for all time t . But this runs in contradiction to Equation 14, as now

$$C_t = Y_t + B_{t-1} \geq Y_t \quad (17)$$

Whenever $G_t > 0$, this causes contradiction.

The inevitable conclusion is that in this basic economy, unless government deficit spending is zero ($G_t = 0$), disequilibrium is unavoidable, unless the idea of forced savings is adopted.

1.1 Interpreting government deficit spending as exports

It can easily be seen that G can be replaced with X , exports. Assume that X is exogenously given and there is zero import. (I will save M for representing money quantity.) One can assume that the foreign sector shares the same currency as the domestic sector, and all central banks have money-printing rights, and that the representative agent of each country cannot change its citizenship. Then it is clear that one faces the exactly same accounting problem.

2 Gali (2014)'s review of money-financed deficit spending

The discussion above is important, as this problem is not properly recognized when dealing with money-financed government spending problems. Gali (2014)

[1] does the exactly same analysis as in the above analysis in the classical monetary economy section, with some utility simplification and additions and some further analysis. Mainly, money is introduced into utility, so utility now looks as:

$$u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{M_t^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (18)$$

where M_t is “real” value of money (in Gali (2014), it is M_t/P_t), with budget constraint:

$$C_t + R_t^{-1}B_t + M_t \leq B_{t-1} + W_t N_t + \Pi_t + M_{t-1} \quad (19)$$

But even with this modification, the only extra optimality condition one obtains is:

$$M_t = \left(\frac{C_t^\sigma}{1 - R_t^{-1}} \right)^{1/\nu} \quad (20)$$

By given knowledge and market clearing, C_t and $E_t C_{t+1}$ are known. Thus, R_t is also known. This means M_t is also known. $B_t = 0$ also in “equilibrium.” Let us re-write the budget constraint into equality (as the household does best to maximize its utility):

$$C_t + M_t - M_{t-1} = C_t + S_t = C_t + G_t = Y_t \quad (21)$$

Thus, $G_t = M_t - M_{t-1}$ must be satisfied. But notice again that C_t and R_t are determined independently of M_t . Suppose that it was found that $G_t = M_t - M_{t-1}$. Then one can adjust ν to make this equality to be untrue, given that the path of G_t remains the same as before.

2.0.1 Fiscal theory of real money supply?

In some ways, these results suggest that some form of fiscal theory of real money supply (here, M_t) is needed to properly form an equilibrium - that the current money-financed deficit spending defines the change in real money supply ($M_t - M_{t-1}$). If this were true, then central banks, by setting nominal money supply $M_t P_t$ defines price level P_t . In a way, this is similar to fiscal theory of price level.

Intuitively, the theory does make sense. After all, G_t is assumed to be all money-financed and this all adds up to real money supply. The problem rather here is why it is the only change possible in net aggregate. Though explaining this constraint may reveal how price level is affected by government spending as equilibrium adjustments.

Also, if one replaces G with X , then the theory converts to current account (CA) surplus/deficit theory of real money supply. One can try to combine two as external surplus/deficit theory of real money supply. But whether this theory is plausible would be left as a question. Notice that the form of a theory can change depending on how utility is specified, so $G_t = M_t - M_{t-1}$ does not always come out as a constraint.

3 Debt-financed deficit spending via bonds, with interest money-financed

So far, in equilibrium $B_t = 0$. Suppose that the government finances its deficit spending G_t by bonds, so $G_t = R_t^{-1}B_t$, if there are equivalent demands, and finance interest by printing money. Again, however, the household has zero demand on B_t . Thus to form an equilibrium properly without G_t constrained to zero or to adopt a non-orthodox theory of money supply, let us introduce B_t into utility.

$$u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{B_t^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad (22)$$

with the previous budget constraint:

$$C_t + R_t^{-1}B_t \leq B_{t-1} + W_t N_t + \Pi_t$$

Here, I drop M_t from utility. But the optimality conditions of the household do change significantly as follows:

$$B_t^{-\nu} - C_t^{-\sigma} R_t^{-1} + \beta E_t [C_{t+1}^{-\sigma}] = 0 \quad (23)$$

Other optimality conditions remain the same. Notice that C_t and C_{t+1} are unaffected by the changed optimality condition. The affected is R_t , and the below is the log-linearized approximation of r_t :

$$r_t = \frac{\sigma(E_t[c_{t+1}] - c_t) + \rho + \nu g_t}{1 - \nu} \quad (24)$$

If g_t is replaced with x_t , then the foreign sector is buying the goods in the domestic sector and selling B_t that the domestic sector willingly takes. Without further restriction, it is certainly possible that the domestic sector continuously buys B_t at all time t that the foreign sector wishes to sell to finance x_t (for the foreign sector this is import). Thus, CA deficits go without the problem in this economy, though this certainly is only theoretical.

The inclusion of B_t in utility results in a different conclusion of welfare effects of fiscal deficit, but I will not explore this question.

References

- [1] Gali, J (2014), "The Effects of a Money-Financed Fiscal Stimulus", CEPR Discussion Paper 10165, September.